Mathematics Areas of Focus: Grade 4

Mission: *Through mathematics, students communicate, make connections, reason, and represent the world quantitatively in order to pose and solve problems.*

Standard 4.1 Number and Numerical Operations

All students will develop number sense and will perform standard numerical operations and estimations on all types of numbers in a variety of ways.

Big Idea: Numeric reasoning involves fluency and facility with numbers.

4.1.4 A. Number Sense

Descriptive Statement: *Number sense is an intuitive feel for numbers and a common sense approach to using them. It is a comfort with what numbers represent that comes from investigating their characteristics and using them in diverse situations. It involves an understanding of how different types of numbers, such as fractions and decimals, are related to each other, and how each can best be used to describe a particular situation. It subsumes the more traditional category of school mathematics curriculum called numeration and thus includes the important concepts of place value, number base, magnitude, and approximation and estimation.*

Essential Questions	Enduring Understandings
▪ How do mathematical ideas interconnect and build on one another to produce a coherent whole? (4.5C1; 4.5C6)** ▪ How can we compare and contrast numbers? (4.5A4)** ▪ How can counting, measuring, or labeling help to make sense of the world around us?	▪ One representation may sometimes be more helpful than another; and, used together, multiple representations give a fuller understanding of a problem. ▪ A quantity can be represented numerically in various ways. Problem solving depends upon choosing wise ways. ▪ Numeric fluency includes both the understanding of and the ability to appropriately use numbers.

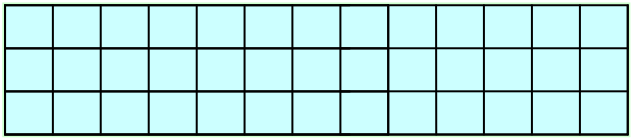
Areas of Focus	Comments and Examples
1. **Use real-life experiences, physical materials, and technology to construct meanings for numbers (unless otherwise noted, all indicators for grade 4 pertain to these sets of numbers as well).** ▪ **Whole numbers through millions** ▪ **Commonly used fractions (denominators of 2, 3, 4, 5, 6, 8, 10, 12, and 16) as part of a whole, as a subset of a set, and as a location on a number line** ▪ **Decimals through hundredths**	**Instructional/Assessment Focus:** • It is important to note that the sets of numbers specified in this CPI also apply to the other grade 4 mathematics CPIs (e.g., 4.1.4A3 and 4.1.4A6 below). **Sample Assessment Item:** • **Extended Constructed Response (ECR):** A class of 24 students will perform an act for the spring talent show. In the class, 2/3 of the students want to perform a skit. The rest of the students want to sing a song. The teacher decided that 3/4 of the students must agree on an act before the decision will be final. ▪ How many of the students want to perform a skit? ▪ How many more students would have to choose a skit before 3/4 of the students agree on it? ▪ Show all of your work and explain your answer. (*Note: Students may draw a picture in response to this question; they are not expected to use formal algorithms for working with fractions at this grade level.*)
2. **Demonstrate an understanding of place value concepts.**	**Sample Assessment Item:** **Multiple Choice (MC):** Using the digits 1 - 7 only once, what is the largest even number you can make with a 5 in the thousands place? a. 7,654,321 b. 7,654,312 * c. 7,645,312 d. 7,435,216
3. **Demonstrate a sense of the relative magnitudes of numbers.**	**Instructional/Assessment Focus:** • Refers not only to whole numbers, but also to fractions and decimals, as specified in 4.1.4A1. **Sample Assessment Item:** ▪ **MC:** If the following fractions were graphed on a number line, which fraction would be closest to zero? a. $\frac{2}{3}$ b. $\frac{1}{4}$ c. $\frac{3}{8}$ * d. $\frac{1}{10}$
4. **Understand the various uses of numbers.** ▪ **Counting, measuring, labeling (e.g., numbers on baseball uniforms), locating (e.g., Room 235 is on the second floor)**	
5. **Use concrete and pictorial models to relate whole numbers, commonly used fractions, and decimals to each other, and to represent equivalent forms of the same number.**	**Sample Assessment Item:** • **SCR:** How many wholes are there in 16/8? _____ *(This item would appear on a non-calculator portion of the statewide assessment. Answer: two or 2)*

6. **Compare and order numbers.**	**Instructional/Assessment Focus:** • Refers not only to whole numbers, but also to fractions and decimals, as specified in 4.1.4A1. **Sample Assessment Item:** ▪ **MC:** Which of the following shows the decimals in order from least to greatest? a. 0.5 0.45 0.54 * b. 0.45 0.5 0.54 c. 0.54 0.5 0.45 d. 0.45 0.54 0.5
7. **Explore settings that give rise to negative numbers.** ▪ **Temperatures below 0^{o}, debts** ▪ **Extension of the number line**	**Instructional/Assessment Focus:** ▪ Students should have the opportunity to explore settings that give rise to negative numbers (e.g., temperatures below 0^{o}, debts, games that involve negative numbers). This would include the use of a thermometer in science experiments. ▪ This content should be introduced at this grade level, but mastery of the content is not assessed in statewide assessment at this grade level.

4.1.4 B. Numerical Operations

Descriptive Statement: *Numerical Operations are an essential part of the mathematics curriculum, especially in the elementary grades. Students must be able to select and apply various computational methods, including mental math, pencil-and-paper techniques, and the use of calculators. Students must understand how to add, subtract, multiply, and divide whole numbers, fractions, decimals, and other kinds of numbers. With the availability of calculators that perform these operations quickly and accurately, the instructional emphasis now is on understanding the meanings and uses of these operations, and on estimation and mental skills, rather than solely on the development of paper-and-pencil proficiency.*

Essential Questions	Enduring Understandings
▪ What makes a computational strategy both effective and efficient? (4.5D1)**	▪ Computational fluency includes understanding not only the meaning, but also the appropriate use of numerical operations.
▪ How do operations affect numbers?	▪ The magnitude of numbers affects the outcome of operations on them.
▪ How do mathematical representations reflect the needs of society across cultures? (An essential question with broad applicability across multiple standards) (4.5C5)**	▪ In many cases, there are multiple algorithms for finding a mathematical solution, and those algorithms are frequently associated with different cultures.

Areas of Focus	Comments and Examples
1. Develop the meanings of the four basic arithmetic operations by modeling and discussing a large variety of problems. ▪ **Addition and subtraction: joining, separating, comparing** ▪ **Multiplication: repeated addition, area/array** ▪ **Division: repeated subtraction, sharing**	**Sample Assessment Items:** • **ECR:** Maria is making apple pies for a party. She bought 3 bags of apples. Each bag has 12 apples. She needs 8 apples to make each pie. ◦ What is the greatest number of pies Maria can make? Show your work or explain your answer. ◦ How many more bags of apples does Maria need to buy in order to make a total of 6 pies? Show your work or explain your answer. • **ECR:** Roxanna built this rectangular array using 39 tiles.  ◦ List two number sentences this model represents. ◦ Roxanna found one more tile. Draw a new rectangular model using all of Roxanna's tiles. ◦ List two multiplication number sentences this new model represents. • **MC:** At West Elementary School, there are 20 more girls than boys. If there are 180 girls, how can you find the number of boys? a. Add 20 to 180 * b. Subtract 20 from 180 c. Multiply 180 by 20 d. Divide 180 by 20

2. Develop proficiency with basic multiplication and division number facts using a variety of fact strategies (such as "skip counting" and "repeated subtraction") and then commit them to memory.	**Sample Assessment Item:** • **MC:** Mrs. Kinney bought batteries in packs of 4 for the students' science experiments. Which of these could be the total number of batteries that she bought? a. 22 b. 26 * c. 28 d. 30
3. Construct, use, and explain procedures for performing whole number calculations with: ▪ **Pencil-and-paper** ▪ **Mental math** ▪ **Calculator**	**Sample Assessment Items:** • **MC:** Find the exact value of 568 ÷ 4. a. 564 * b. 142 c. 140 d. 112 *(This item would appear on a non-calculator portion of the statewide assessment.)* • **SCR:** Find the exact answer: 568 ÷ 4 = _____ *(This item would appear on a non-calculator portion of the statewide assessment. Answer: 142)* • **SCR:** Find the exact answer: 4 x 25 x 9 = _____ *(This item would appear on a non-calculator portion of the statewide assessment. Answer: 900)* • **SCR:** Find the exact answer: 800 - 301 = _____ *(This item would appear on a non-calculator portion of the statewide assessment. Answer: 499)*
4. Use efficient and accurate pencil-and-paper procedures for computation with whole numbers. ▪ Addition of 3-digit numbers ▪ Subtraction of 3-digit numbers ▪ Multiplication of 2-digit numbers ▪ Division of 3-digit numbers by 1-digit numbers	**Sample Assessment Items:** **MC:** 20 x 70 = a. 14 b. 140 * c. 1,400 d. 14,000 *(This item would appear on a non-calculator portion of the statewide assessment.)* **MC:** 810 - 18 = a. 828 b. 808 c. 802 * d. 792 *(This item would appear on a non-calculator portion of the statewide assessment.)* **MC:** 56 X 74 a. 120 b. 130 c. 3,144 * d. 4,144 *(This item would appear on a non-calculator portion of the statewide assessment.)* • **SCR:** If 942 trading cards are divided equally among 3 students, how many trading cards would each receive? _____ *(This item would appear on a non-calculator portion of the statewide assessment. Answer: 314 trading cards)*
5. Construct and use procedures for performing decimal addition and subtraction.	**Instructional/Assessment Focus:** • This content should be introduced at this grade level, with decimals through hundredths (as specified in 4.1.4A1), but statewide assessment of the content is limited at this grade level. Much of the assessment of this CPI will be within the context of CPI 4.1.4B6.
6. Count and perform simple computations with money. ▪ **Standard dollars and cents notation**	**Sample Assessment Item:** • **ECR:** Sarah goes to the store to buy some food for an afternoon snack. She buys a bottle of orange juice for $1.67, a bag of pretzels for $0.89, and 2 apples for $0.45 each. She must also pay $0.16 tax. • How much does Sarah have to pay in all? Show your work. • What bills and coins would Sarah give to the salesperson to pay for the food using exact change?
7. Select pencil-and-paper, mental math, or a calculator as the appropriate computational method in a given situation depending on the context and numbers.	**Sample Assessment Item:** • **Sample MC Item:** Find the exact answer: 4 x 25 x 9 = a. 90 b. 100 c. 360 * d. 900 *(This item would appear on a non-calculator portion of the statewide assessment.)*
8. Check the reasonableness of results of computations.	**Instructional/Assessment Focus:** • Includes: o identifying unreasonable answers obtained using a calculator; o the use of inverse operations to check solutions; o reasoning (4.5D2) and communication (4.5B2)**; o solving problems (4.5A2)** involving this recognition; and • Is most applicable to whole numbers at this grade level, rather than to fractions or decimals.

9. Use concrete models to explore addition and subtraction with fractions.	**Instructional/Assessment Focus:** • The intent at grade 4 is that students be provided with opportunities to develop a better conceptual understanding of fractions and non-algorithmic addition and subtraction using visual models (either physical or electronic). Formal algorithmic procedures for adding and subtracting fractions are an area of focus in grade 5.
10. Understand and use the inverse relationships between addition and subtraction and between multiplication and division.	"Use" here means "apply." **Assessment Focus:** The emphasis in statewide assessment is on the "use" or "apply," rather than on the "understand."

4.1.4 C. Estimation

Descriptive Statement: *Estimation is a process that is used constantly by mathematically capable adults, and one that can be easily mastered by children. It involves an educated guess about a quantity or an intelligent prediction of the outcome of a computation. The growing use of calculators makes it more important than ever that students know when a computed answer is reasonable; the best way to make that determination is through the use of strong estimation skills. Equally important is an awareness of the many situations in which an approximate answer is as good as, or even preferable to, an exact one. Students can learn to make these judgments and use mathematics more powerfully as a result.*

Essential Questions	Enduring Understandings
▪ How can we decide when to use an exact answer and when to use an estimate?	▪ Context is critical when using estimation.

Areas of Focus	Comments and Examples
1. Judge without counting whether a set of objects has less than, more than, or the same number of objects as a reference set.	**Instructional/Assessment Focus:** • This is an area of focus in grade 3 and may be assessed at a higher level of understanding in grade 4.
2. Construct and use a variety of estimation strategies (e.g., rounding and mental math) for estimating both quantities and the result of computations.	**Instructional/Assessment Focus:** • An area of focus in grade 3 for whole-number addition and subtraction, this CPI is an area of focus in grade 4 for whole-number multiplication and division and also for addition and subtraction of decimals. **Sample Assessment Items:** • **MC:** Estimate 39 X 11. The product is between which numbers? a. 30 and 80 * b. 300 and 800 c. 3,000 and 8,000 d. 30,000 and 80,000 *(This item would appear on a non-calculator portion of the statewide assessment.)* • **MC:** Estimate $756 \div 8$. The quotient is between which numbers? a. 8 and 10 b. 11 and 13 c. 80 and 100 d. 110 and 130 *(This item would appear on a non-calculator portion of the statewide assessment.)*
3. **Recognize when an estimate is appropriate, and understand the usefulness of an estimate as distinct from an exact answer.**	**Instructional/Assessment Focus:** • Assessment of this CPI and demonstration of this understanding is frequently within the context of one or more of the other content CPIs. **Sample Assessment Items:** • **MC:** For which of the following would it generally be better to calculate the exact answer than to estimate? a. The number of words in a composition * b. The number of runs scored by a team in a baseball game c. The number of steps taken on your way to school d. The number of miles you traveled on your vacation • **MC:** In which of the following situations would it be better to estimate than to calculate the exact answer? a. To feed your family hamburgers, you need the number of family members * b. To purchase paint for a wall, you want the area of the wall in square feet c. To give a customer change, you want the cost of the items purchased d. To buy theater tickets, you want the number of people attending the show
4. **Use estimation to determine whether the result of a computation (either by calculator or by hand) is reasonable.**	**Sample Assessment Items:** • **ECR:** Kelly predicted that each of the 24 fourth grade students in her class would use 52 sheets of composition paper during the coming month. Sam told Kelly that 24 x 52 = 2284. Use estimation to explain if you think Sam is right or wrong and why. • **ECR:** John had $4.70 to purchase a binder. However, he found a cheaper binder in the store for $3.27. Amy told John that $4.70 – $3.27 = $2.20. Use estimation to explain why you think Amy is right or wrong.

Standard 4.2 Geometry and Measurement

All students will develop spatial sense and the ability to use geometric properties, relationships, and measurement to model, describe and analyze phenomena.

Big Idea *Geometry***:** Spatial sense and geometric relationships are a means to solve problems and make sense of a variety of phenomena.

Big Idea *Measurement***:** Measurement is a tool to quantify a variety of phenomena.

4.2.4 A. Geometric Properties

Descriptive Statement: *This includes identifying, describing and classifying standard geometric objects, describing and comparing properties of geometric objects, making conjectures concerning them, and using reasoning and proof to verify or refute conjectures and theorems. Also included here are such concepts as symmetry, congruence, and similarity.*

Essential Questions	Enduring Understandings
▪ How can spatial relationships be described by careful use of geometric language? ▪ How do geometric relationships help us to solve problems and/or make sense of phenomena?	▪ Geometric properties can be used to construct geometric figures. (4.5D1; 4.5D2; 4.5E3)** ▪ Geometric relationships provide a means to make sense of a variety of phenomena.

Areas of Focus

1. Identify and describe spatial relationships of two or more objects in space.
- **Direction, orientation, and perspectives (e.g., which object is on your left when you are standing here?)**
- **Relative shapes and sizes**
- **Shadows (projections) of everyday objects**

Comments and Examples

Sample Assessment Items:

- **MC:** There are 7 desks arranged in a row in Mr. Thompson's classroom. Hector sits 2 seats to the right of Kim. Tonya sits 3 seats to the right of Hector.
  How many seats to the left of Tonya does Kim sit?
  a. 2 b. 3 * c. 5 d. 12
- **MC:** Which of the following shadows can not be made by a cylinder?

a. b. * c. d.

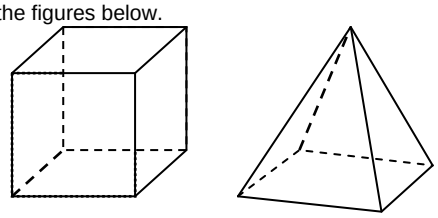
2. Use properties of standard three-dimensional and two-dimensional shapes to identify, classify, and describe them.
- **Vertex, edge, face, side, angle**
- **3D figures – cube, rectangular prism, sphere, cone, cylinder, and pyramid**
- **2D figures – square, rectangle, circle, triangle, quadrilateral, pentagon, hexagon, octagon**
- **Inclusive relationships – squares are rectangles, cubes are rectangular prisms**

Instructional/Assessment Focus:

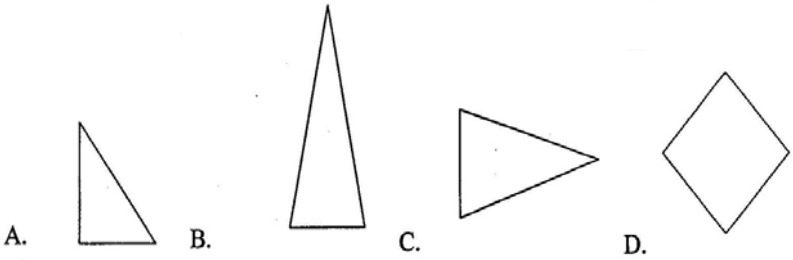
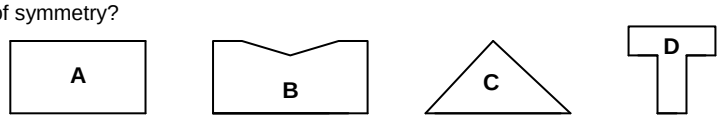
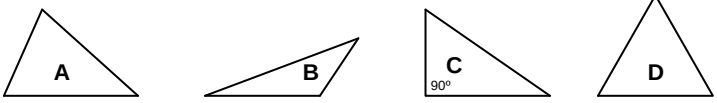
- This CPI includes some shapes and concepts that were introduced in grade 3, and then adds quadrilaterals and inclusive relationships.

Sample Assessment Items:

- **MC:** I have 2 faces, no vertices, and I can roll.
  What am I?
  a. cone * b. cylinder c. sphere d. prism
- **MC:** Which of these figures has opposite sides parallel and 4 right angles?
  * a. rectangle b. triangle c. hexagon d. octagon
- **ECR:** Look at the figures below.



  - Name each figure.
  - How many faces does each figure have?
  - Write one way the figures are the same.
  - Write one way the figures are different.

3. Identify and describe relationships among two-dimensional shapes. ▪ **Congruence** ▪ **Lines of symmetry**	**Sample Assessment Items:** • **MC:** Which figure is congruent to Shape T?  Shape T  A. B. C. D. a. A b. B * c. C d. D • **MC:** Neil made these cutouts for a mobile. Which cutout has more than one line of symmetry?  * a. A b. B c. C d. D
4. Understand and apply concepts involving lines, angles, and circles. ▪ **Point, line, line segment, endpoint** ▪ **Parallel, perpendicular** ▪ **Angles – acute, right, obtuse** ▪ **Circles – diameter, radius, center**	"Understand and apply" here means "define, recognize, and apply." **Sample Assessment Item:** • **MC:** Which triangle has an angle larger than a right angle?  a. **A** * b. **B** c. **C** d. **D** (*Note: At this grade level, the question might also be worded, "Which triangle has an obtuse angle?*)
5. Recognize, describe, extend, and create space-filling patterns.	**Instructional/Assessment Focus:** • This is an area of focus in grade 3 and may be assessed at a higher level of understanding in grade 4.

4.2.4 B. Transforming Shapes

Descriptive Statement: *This includes identifying, describing and classifying standard geometric objects, describing and comparing properties of geometric objects, making conjectures concerning them, and using reasoning and proof to verify or refute conjectures and theorems. Also included here are such concepts as symmetry, congruence, and similarity.*

Essential Questions	Enduring Understandings
▪ What situations can be analyzed using transformations and symmetries? (4.5E1; 4.5E2; 4.5E3)**	▪ Shape and area can be conserved during mathematical transformations.
Areas of Focus	**Comments and Examples**

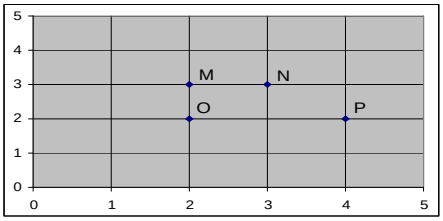
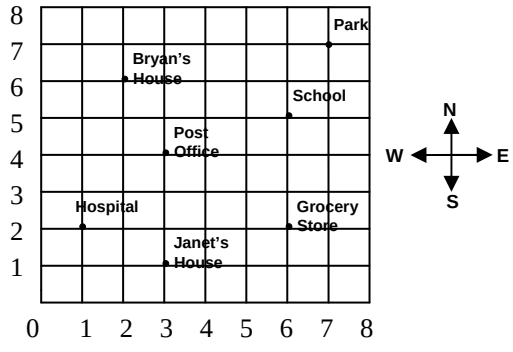
1. Use simple shapes to cover an area (tessellations).	**Suggested Instructional/Assessment Strategies:**
	• This content provides an opportunity to integrate mathematics with the visual arts. Students can:
	  • view prints by M.C. Escher and see how tessellations can become a famous art form;
	  • engage in problem solving as they discover the different ways they can tessellate polygons from pattern blocks or geoblocks;
	  • tessellate shapes using slides, rotations, and reflections; or
	  • explore various figures (including, but not limited to, those mentioned in CPIs 4.2.3A2 and 4.2.4A2) as they try to tessellate kites, ovals, parallelograms, rhombi, triangles, pentagons, hexagons, circles, or rectangles.

2. Describe and use geometric transformations (slide, flip, turn).	**Instructional/Assessment Focus:** • This is an area of focus in grade 3 and may be assessed at a higher level of understanding in grade 4.
3. Investigate the occurrence of geometry in nature and art.	**Instructional/Assessment Focus:** • This is an area of focus in grade 3 and may be assessed at a higher level of understanding in grade 4.

4.2.4 C. Coordinate Geometry

Descriptive Statement: *Coordinate geometry provides an important connection between geometry and algebra. It facilitates the visualization of algebraic relationships, as well as an analytical understanding of geometry.*

Essential Questions	Enduring Understandings
▪ How can we best represent and verify geometric/algebraic relationships? (4.5C2; 4.5D2; 4.5E1; 4.5E2; 4.5F5)**	▪ Reasoning and/or proof can be used to verify or refute conjectures or theorems in geometry (4.5D1; 4.5D3; 4.5D4; 4.5D5; 4.5F5)** ▪ Coordinate geometry can be used to represent and verify geometric/algebraic relationships.

Areas of Focus	Comments and Examples
1. Locate and name points in the first quadrant on a coordinate grid.	**Instructional/Assessment Focus:** • This is an area of focus in grade 3 and may be assessed at a higher level of understanding in grade 4. **Sample Assessment Item:** • **MC:** What is the location of point M?  *a. (2, 2)* ** b. (2, 3)* *c. (3, 3)* *d. (4, 2)*
2. Use coordinates to give or follow directions from one point to another on a map or grid.	**Suggested Instructional/Assessment Strategy:** • Note the connections with CPI 4.2.4A1 (orientation) and CPI 4.4.4D1 (directions). **Sample Assessment Item:** • **ECR:** What directions could you give to someone to get from Janet's house to the school? 

4.2.4 D. Units Of Measurement

Descriptive Statement: *Measurement helps describe our world using numbers. An understanding of how we attach numbers to real-world phenomena, familiarity with common measurement units (e.g., inches, liters, and miles per hour), and a practical knowledge of measurement tools and techniques are critical for students' understanding of the world around them.*

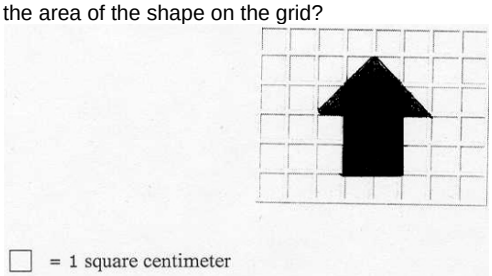
Essential Questions	Enduring Understandings
▪ How can measurements be used to solve problems? (4.5A6)**	▪ Everyday objects have a variety of attributes, each of which can be measured in many ways. ▪ What we measure affects how we measure it. (4.5A4; 4.5A6)** ▪ Measurements can be used to describe, compare, and make sense of phenomena.

Areas of Focus	Comments and Examples
1. Understand that everyday objects have a variety of attributes, each of which can be measured in many ways.	**Instructional/Assessment Focus:** • This is an area of focus in grade 3 and may be assessed at a higher level of understanding in grade 4.
2. Select and use appropriate standard units of measure and measurement tools to solve real-life problems. ▪ **Length – fractions of an inch (1/8, 1/4, 1/2), mile, decimeter, kilometer** ▪ **Area – square inch, square centimeter** ▪ **Volume – cubic inch, cubic centimeter** ▪ **Weight – ounce** ▪ **Capacity – fluid ounce, cup, gallon, milliliter**	**Sample Assessment Item:** • **MC:** What is the most reasonable estimate of the length of a city's swimming pool? a. 1 meter * b. 25 meters c. 1 kilometer d. 25 kilometers
3. Develop and use personal referents to approximate standard units of measure (e.g., a common paper clip is about an inch long).	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI. Assessment of this CPI is generally within the context of one or more of the other content CPIs. **Suggested Instructional/Assessment Strategy:** • Students identify parts of their body that are the same length as 10 centimeters and use them to measure the length of their pencil.
4. Incorporate estimation in measurement activities (e.g., estimate before measuring).	**Instructional/Assessment Focus:** • This is an area of focus in grade 3 and may be assessed at a higher level of understanding in grade 4.
5. Solve problems involving elapsed time.	**Sample Assessment Item:** • **MC:** Michelle went to her friend's house at 1:15 P.M. Her father told her to be home in 1 hour and 45 minutes. What time did Michelle need to be home?  a. 2:00 P.M. b. 2:30 P.M. c. 2:45 P.M. * d. 3:00 P.M.

4.2.4 E. Measuring Geometric Objects

Descriptive Statement: *This area focuses on applying the knowledge and understandings of units of measurement in order to actually perform measurement. While students will eventually apply formulas, it is important they develop and apply strategies that derive from their understanding of the attributes. In addition to measuring objects directly, students apply indirect measurement skills, using, for example, similar triangles and trigonometry.*

Essential Questions	Enduring Understandings
▪ How can measurements be used to solve problems? (4.5A6)**	▪ Everyday objects have a variety of attributes, each of which can be measured in many ways. ▪ What we measure affects how we measure it. (4.5A4; 4.5A6)** ▪ Measurements can be used to describe, compare, and make sense of phenomena.

Areas of Focus	Comments and Examples
1. Determine the area of simple two-dimensional shapes on a square grid.	**Sample Assessment Item:** • **MC:** What is the area of the shape on the grid?  a. 6 square centimeters * b. 8 square centimeters c. 10 square centimeters d. 12 square centimeters

2. Distinguish between perimeter and area and use each appropriately in problem-solving situations.	**Sample Assessment Item:**
	• **ECR:** Veronica is making a rectangular garden. She plans to put a fence around the garden using 28 feet of fencing, and she wants the garden to be 8 feet long.
	  • How wide will Veronica's garden be? Show your work or explain how you got your answer.
	  • If Veronica is going to put fence posts two feet apart around the outside of the garden, how many fence posts will she need? Show your work or explain your answer.

3. Measure and compare the volume of three–dimensional objects using materials such as rice or cubes.	**Instructional/Assessment Focus:** • Students are expected to solve problems (4.5A2)** involving this recognition. • While the emphasis in grade 3 was be on the "measure," the emphasis in grade 4 is on the "compare."

Standard 4.3 Patterns and Algebra

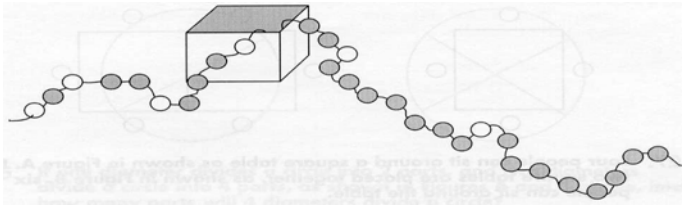
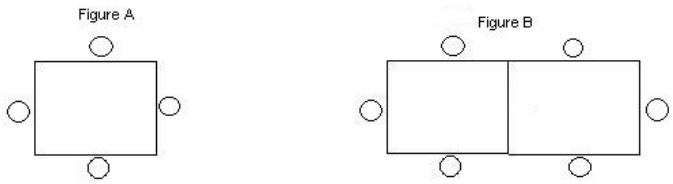
All students will represent and analyze relationships among variable quantities and solve problems involving patterns, functions, and algebraic concepts and processes.

Big Idea: Algebra provides language through which we communicate the patterns in mathematics.

4.3.4 A. Patterns

Descriptive Statement: *Algebra provides the language through which we communicate the patterns in mathematics. From the earliest age, students should be encouraged to investigate the patterns that they find in numbers, shapes, and expressions, and by doing so, to make mathematical discoveries. They should have opportunities to analyze, extend, and create a variety of patterns and to use pattern-based thinking to understand and represent mathematical and other real-world phenomena.*

Essential Questions	Enduring Understandings
▪ How can change be best represented mathematically? (4.5C1; 4.5F1; 4.5F2; 4.5F3; 4.5F4)** ▪ How can patterns, relations, and functions be used as tools to best describe and help explain real-life situations? (4.5C1)**	▪ The symbolic language of algebra is used to communicate and generalize the patterns in mathematics. ▪ Algebraic representation can be used to generalize patterns and relationships.

Areas of Focus	Comments and Examples
1. Recognize, describe, extend, and create patterns. ▪ **Descriptions using words, number sentences/expressions, graphs, tables, variables (e.g., shape, blank, or letter)** ▪ **Sequences that stop or that continue infinitely** ▪ **Whole number patterns that grow or shrink as a result of repeatedly adding, subtracting, multiplying by, or dividing by a fixed number (e.g., 5, 8, 11, . . . or 800, 400, 200, . . .)** ▪ **Sequences can often be extended in more than one way (e.g., the next term after 1, 2, 4, . . . could be 8, or 7, or …)**	**Sample Assessment Items:** • **MC:** If this pattern continues, what is the next number? 5, 8, 7, 10, 9, 12, 11, … * a. 14 b. 13 c. 12 d. 10 • ***ECR:***  Sally is making a necklace by stringing beads in a certain pattern. A section of the unfinished necklace is inside the box. Based on the pattern shown, draw or describe in detail the section of the necklace that is inside the box. Show your work and clearly explain your answer. • **ECR:**  Four people can sit around a square table as shown in Figure A. If two square tables are placed together, as shown in Figure B, six people can sit around the table. If six of these tables are placed together in a single row to make one long rectangular table, how many people would be able to sit around it? a. 13 * b. 14 c. 19 d. 24

4.3.4 B. Functions & Relationships

Descriptive Statement: *The function concept is one of the most fundamental unifying ideas of modern mathematics. Student begin their study of functions in the primary grades, as they observe and study patterns. As students grow and their ability to abstract matures, students form rules, display information in a table or chart, and write equations which express the relationships they have observed. In high school, they use the more formal language of algebra to describe these relationships.*

Essential Questions	Enduring Understandings
▪ How are patterns of change related to the behavior of functions? (4.5F1; 4.5F2; 4.5F3; 4.5F4)**	▪ Patterns and relationships can be represented graphically, numerically, symbolically, or verbally. (4.5E1)**

Areas of Focus

1. Use concrete and pictorial models to explore the basic concept of a function.
- **Input/output tables, T-charts**
- **Combining two function machines**
- **Reversing a function machine**

Comments and Examples

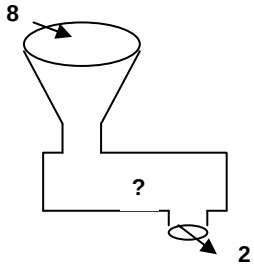
Sample Assessment Items:

- **MC:** What number is missing in the output column of the table below?

Input	Output
4	12
6	18
7	
9	27

a. 20 * b. 21 c. 22 d. 24

- **MC Item:** When 8 is dropped into this machine, it comes out as 2.



The table shows some other input and output data for the machine.

Input	8	10	15	12
Output	2	4	9	

What is the missing number in the table?

a. 5 * b. 6 c. 10 d. 11

- **ECR:** You are trying to save money to buy a present for your friend. You record the total amount of money you have at the end of each week.

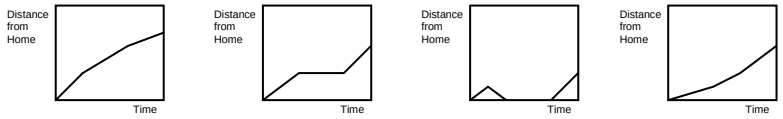
Week	1	2	3	4	5
Total amount of money saved	$1.75	$3.50	$5.25	$7.00	

  - If you continue saving money following this pattern, how much money will you have at the end of week 5? Explain the pattern you used to get your answer.
  - The gift you would like to buy costs $12.00. How many weeks will it take you to save at least that much money? Show your work and explain your answer.

4.3.4 C. Modeling

Descriptive Statement: *Algebra is used to model real situations and answer questions about them. This use of algebra requires the ability to represent data in tables, pictures, graphs, equations or inequalities, and rules. Modeling ranges from writing simple number sentences to help solve story problems in the primary grades to using functions to describe the relationship between two variables, such as the height of a pitched ball over time. Modeling also includes some of the conceptual building blocks of calculus, such as how quantities change over time and what happens in the long run (limits).*

Essential Questions	Enduring Understandings
▪ How can we use mathematical models to describe physical relationships? (4.5E2)**	▪ Mathematical models can be used to describe and quantify physical relationships. (4.5E2)**
▪ How can we use physical models to clarify mathematical relationships? (4.5E3)**	▪ Physical models can be used to clarify mathematical relationships. (4.5E3)**

Areas of Focus	Comments and Examples
1. Recognize and describe change in quantities. ▪ **Graphs representing change over time (e.g., temperature, height)** ▪ **How change in one physical quantity can produce a corresponding change in another (e.g., pitch of a sound depends on the rate of vibration)**	**Sample Assessment Items:** • **MC:** The temperature in the food freezer was 70ºF when it was plugged in at 12:00 noon. If the temperature went down 8ºF every hour, what was the temperature in the freezer at 3:00 P.M.? a. 62ºF b. 54ºF * c. 46ºF d. 28ºF • **MC:** We left home to drive to the shore. Things went fine until we had a flat tire. Which graph goes with this story? a. * b. c. d. 
2. Construct and solve simple open sentences involving any one operation (e.g., 3 x 6 = __, n = 15 ÷ 3, 3 x __ = 0, 16 – c = 7).	**Sample Assessment Items:** • **MC**: If ❒ x 8 = 96, what is the value of ❒? * a. 12 b. 88 c. 104 d. 768 • **MC:** If 84 ÷ ❒ = 7, then what is the value of ❒? a. 91 b. 77 * c. 12 d. 7 • **MC:** Some very old books do not have the pages numbered. Mrs. Jensen is a librarian and has developed a rule for estimating the number of pages (P), given the weight of the book. Which of the following is most likely the rule Mrs. Jensen uses for a book weighing two pounds? * a. P = 100 x 2 b. P = 100 - 2 c. P = 100 + 2 d. P = 100 ÷ 2

4.3.4 D. Procedures

Descriptive Statement: *Techniques for manipulating algebraic expressions - procedures - remain important, especially for students who may continue their study of mathematics in a calculus program. Utilization of algebraic procedures includes understanding and applying properties of numbers and operations, using symbols and variables appropriately, working with expressions, equations, and inequalities, and solving equations and inequalities.*

Essential Questions	Enduring Understandings
▪ What makes an algebraic algorithm both effective and efficient? (4.5D1)**	▪ Algebraic and numeric procedures are interconnected and build on one another to produce a coherent whole. ▪ Reasoning and/or proof can be used to verify or refute conjectures or theorems in algebra. (4.5D1; 4.5D3; 4.5D4; 4.5D5)**

Areas of Focus	Comments and Examples
1. Understand, name, and apply the properties of operations and numbers. ▪ **Commutative (e.g., 3 x 7 = 7 x 3)** ▪ **Identity element for multiplication is 1 (e.g., 1 x 8 = 8)** ▪ **Associative (e.g., 2 x 4 x 25 can be found by first multiplying either 2 x 4 or 4 x 25)** ▪ **Division by zero is undefined** ▪ **Any number multiplied by zero is zero**	**Sample Assessment Items:** • **MC**: Which expression gives the same result as 2 x 4 x 25? a. 2 x 9 x 5 b. 6 x 25 * c. 2 x 100 d. 4 x 27 • **ECR:** Sue thinks that 3/0 is 3. • Is she correct? • Explain why you believe she is correct or incorrect.
2. Understand and use the concepts of equals, less than, and greater than in simple number sentences. ▪ **Symbols (= , < , >)**	**Sample Assessment Item:** • **MC:** Allison and Michele each had $5.00 to spend at the bookstore. Allison bought a book that cost $3.50 and Michele bought a book that cost $4.25. Which of the following correctly compares the amount of money each girl has left? a. $5.00 - $3.50 < $5.00 - $4.25 b. $5.00 - $4.25 > $5.00 - $3.50 * c. $5.00 - $3.50 > $5.00 - $4.25 d. $5.00 - $4.25 = $5.00 - $3.50

Standard 4.4 Data Analysis, Probability, and Discrete Mathematics

All students will develop an understanding of the concepts and techniques of data analysis, probability, and discrete mathematics, and will use them to model situations, solve problems, and analyze and draw appropriate inferences from data.

Big Idea *Data Analysis*: Reading, understanding, interpreting, and communicating data are critical in modeling a variety of real-world situations, drawing appropriate inferences, making informed decisions, and justifying those decisions.

Big Idea *Probability*: Probability quantifies the likelihood that something will happen and enables us to make predictions and informed decisions.

Big Idea *Discrete Mathematics*: Discrete mathematics consists of tools and strategies for representing, organizing, and interpreting non-continuous data.

4.4.4 A. Data Analysis

Descriptive Statement: *In today's information-based world, students need to be able to read, understand, and interpret data in order to make informed decisions. In the early grades, students should be involved in collecting and organizing data, and in presenting it using tables, charts, and graphs. As they progress, they should gather data using sampling, and should increasingly be expected to analyze and make inferences from data, as well as to analyze data and inferences made by others.*

Essential Questions	Enduring Understandings
▪ How can the collection, organization, interpretation, and display of data be used to answer questions? (4.5A4; 4.5A6; 4.5E1; 4.5E2; 4.5F1; 4.5F6)**	▪ The message conveyed by the data depends on how the data is collected, represented, and summarized. (4.5A6; 4.5D6; 4.5E1; 4.5E2; 4.5E3)** ▪ The results of a statistical investigation can be used to support or refute an argument. (4.5D1; 4.5D3; 4.5D5; 4.5E2; 4.5E3; 4.5F6)**

Areas of Focus

1. Collect, generate, organize, and display data in response to questions, claims, or curiosity.
- Data collected from the school environment

2. **Read, interpret, construct, analyze, generate questions about, and draw inferences from displays of data.**
- **Pictograph, bar graph, line plot, line graph, table**
- **Average (mean), most frequent (mode), middle term (median)**

Comments and Examples

Instructional/Assessment Focus:
- Assessment will focus on organization and display of data, more than the collecting or generating of data. The actual gathering of data may appropriately receive additional attention during instruction.
- Assessment of this CPI is frequently within the context of CPI 4.4.4A2.

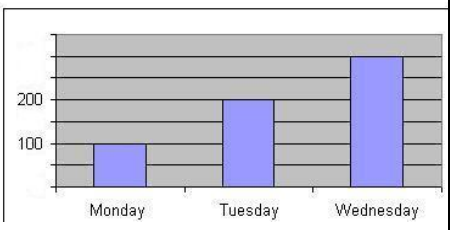
Sample Assessment Items:

- **MC:** The newspaper reported the following temperatures for the city of Trenton for a certain week in May.

DAY	Temperature (in degrees Fahrenheit)
Monday	75°
Tuesday	69°
Wednesday	68°
Thursday	62°
Friday	58°
Saturday	72°
Sunday	72°

What was the average (mean) temperature of the week?
*a. 58° *b. 68° c. 69° d. 72°*

- **MC:** The graph to the right shows the number of tickets sold for the first three days of the state fair. What is the approximate number of tickets sold?
  a. 450
  b. 500
  c. 550
  * d. 600



- **MC:** Mrs. Barber's class kept track of how many pages they read during the school year. Each ball on the pictograph stands for 50 pages.

Lucas	●●●●●●●●◖
Yolanda	●●●●●●●● ●●●●●●●●
Katie	●●●●●●● ●●●●●●●
Sam	●●●●●● ●●●●●

● = 50 pages

How many pages did Lucas read?
a. 425 b. 450 * c. 475 d. 500

4.4.4 B. Probability

Descriptive Statement: *Students need to understand the fundamental concepts of probability so that they can interpret weather forecasts, avoid unfair games of chance, and make informed decisions about medical treatments whose success rate is provided in terms of percentages. They should regularly be engaged in predicting and determining probabilities, often based on experiments (like flipping a coin 100 times), but eventually based on theoretical discussions of probability that make use of systematic counting strategies. High school students should use probability models and solve problems involving compound events and sampling.*

Essential Questions	Enduring Understandings
▪ How can experimental and theoretical probabilities be used to make predictions or draw conclusions? (4.5D5; 4.5D6)**	▪ Experimental results tend to approach theoretical probabilities after a large number of trials.

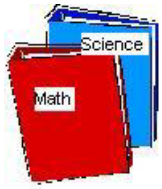
Areas of Focus	Comments and Examples
1. Use everyday events and chance devices, such as dice, coins, and unevenly divided spinners, to explore concepts of probability. **▪ Likely, unlikely, certain, impossible, improbable, fair, unfair** **▪ More likely, less likely, equally likely** **▪ Probability of tossing "heads" does not depend on outcomes of previous tosses**	**Instructional/Assessment Focus:** • The exploration using dice, coins, and unevenly divided spinners is largely instructional, and generally assessed indirectly on statewide assessments. • Familiarity with the concepts and vocabulary in the bullets is frequently assessed within the context of CPI 4.4.4B3.
2. Determine probabilities of simple events based on equally likely outcomes and express them as fractions.	**Instructional/Assessment Focus:** • Assessment of this CPI is generally within the context of CPI 4.4.4B3.
3. Predict probabilities in a variety of situations (e.g., given the number of items of each color in a bag, what is the probability that an item picked will have a particular color). **▪ What students think will happen (intuitive)** **▪ Collect data and use that data to predict the probability (experimental)** **▪ Analyze all possible outcomes to find the probability (theoretical)**	**Instructional/Assessment Focus:** • In fourth grade, students are expected to write probabilities as fractions, although they are not necessarily expected to reduce those fractions until fifth grade. **Sample Assessment Items:** • **SCR:** If there are seven marbles in a bag, three red and four green, what is the probability that a marble picked from the bag will be red? (Answer: 3/7 or 3 out of 7) • **MC:** Cynthia has a bag of 10 marbles that contains 4 red marbles and 6 blue marbles. If Cynthia reached into the bag without looking and picked one marble, what is the probability that she would pick a blue marble? a. 1/10 b. 4/10 * c. 6/10 d. 10/10 • **MC:** Joanne has a bag of marbles that contains 5 blue marbles, 4 red marbles, 2 white marbles, and 1 yellow marble. If Joanne wants to pick a marble out of the bag without looking, what is the probability that she will pick a red or yellow marble? a. $\frac{1}{12}$ b. $\frac{4}{12}$ * c. $\frac{5}{12}$ d. $\frac{7}{12}$

4.4.4 C. Discrete Mathematics - Systematic Listing And Counting

Descriptive Statement: *Development of strategies for listing and counting can progress through all grade levels, with middle and high school students using the strategies to solve problems in probability. Primary students, for example, might find all outfits that can be worn using two coats and three hats; middle school students might systematically list and count the number of routes from one site on a map to another; and high school students might determine the number of three-person delegations that can be selected from their class to visit the mayor.*

Essential Questions	Enduring Understandings
▪ How can attributes be used to classify data/objects?	▪ Grouping by attributes (classification) can be used to answer mathematical questions. (4.5E1; 4.5E3)**
▪ What is the best way to solve this? What counting strategy works best here?	▪ Algorithms can effectively and efficiently be used to quantify and interpret discrete information.

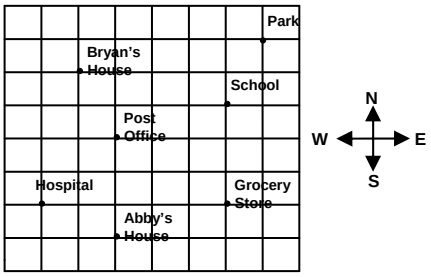
Areas of Focus	Comments and Examples
1. Represent and classify data according to attributes, such as shape or color, and relationships. ▪ Venn diagrams ▪ Numerical and alphabetical order	**Instructional/Assessment Focus:** This is an area of focus in grade 3 and may be assessed at a higher level of understanding in grade 4.

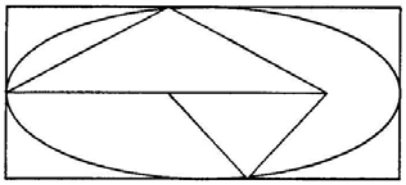
2. Represent all possibilities for a simple counting situation in an organized way and draw conclusions from this representation. ▪ **Organized lists, charts, tree diagrams** ▪ **Dividing into categories (e.g., to find the total number of rectangles in a grid, find the number of rectangles of each size and add the results)**	**Sample Assessment Items:** • **MC:** George has 4 notebooks: one for math, one for science, one for language arts, and one for social studies. He is going to take two notebooks home tonight. How many different pairs of notebooks could he take home? a. 2 b. 4 * c. 6 d. 8  one pair • **ECR:** Jennifer has a new kitten. His name is Buddy. Buddy needs a collar and a bell. Jennifer is looking at collars that come in blue, red, yellow, or green and bells that come in gold or silver. • Show all the different combinations of collars and bells that Jennifer can make for Buddy. Jennifer also wants a name tag for Buddy. The name tag can be large or small. • How many total combinations of collars, bells, and name tags are possible? Show your work or explain your answer.  • **SCR:** A juice machine charges $0.65 for a can of juice and accepts only nickels, dimes, or quarters. The machine requires exact change. You have 4 nickels, 4 dimes, and 4 quarters. Make a table and list the different ways you can use your coins to make up exactly $0.65. Show your work and clearly explain your answer.

4.4.4 D. Discrete Mathematics - Vertex-Edge Graphs And Algorithms

Descriptive Statement: *Vertex-edge graphs, consisting of dots (vertices) and lines joining them (edges), can be used to represent and solve problems based on real-world situations. Students should learn to follow and devise lists of instructions, called "algorithms," and use algorithmic thinking to find the best solution to problems like those involving vertex-edge graphs, but also to solve other problems.*

Essential Questions	Enduring Understandings
▪ How can visual tools such as networks (vertex-edge graphs) be used to answer questions? (4.5E1; 4.5E3)** ▪ How can algorithmic thinking be used to solve problems?	▪ Optimization is finding the best solution within given constraints. ▪ Algorithms can effectively and efficiently be used to quantify and interpret discrete information.

Areas of Focus	Comments and Examples
1. Follow, devise, and describe practical sets of directions (e.g., to add two 2-digit numbers).	**Suggested Instructional/Assessment Strategy:** • Note the connections with CPI 4.2.4A1 (orientation) and CPI 4.2.4C2 (using coordinates to give or follow directions). **Sample Assessment Item:** • **ECR:** Look at the map below.  Abby left her house and followed this list of directions: 1. Walk two blocks west. 2. Walk three blocks north. 3. Walk five blocks east. 4. Walk one block north. • Use the map to help you list all of the places Abby passed on her walk, including the place where she ended her walk. After school, Bryan is going to Abby's house to trade cards. He wants to stop at home first to get his trading cards. • Make a list of directions that Bryan can follow to walk from school to his house and then to Abby's house.

2. Play two-person games and devise strategies for winning the games (e.g., "make 5" where players alternately add 1 or 2 and the person who reaches 5, or another designated number, is the winner).	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI. Assessment of this CPI is generally within the context of one or more of the other content CPIs.
3. Explore vertex-edge graphs and tree diagrams. ▪ Vertex, edge, neighboring/adjacent, number of neighbors ▪ Path, circuit (i.e., path that ends at its starting point)	**Instructional/Assessment Focus:** • This Content should be introduced at this grade level, but mastery of the content is not assessed in statewide assessment at this grade level.
4. Find the smallest number of colors needed to color a map or a graph.	**Instructional/Assessment Focus:** • This is an area of focus in grade 3 and may be assessed at a higher level of understanding in grade 4. **Sample Assessment Item:** • **MC:** To color the following map, you want to use as few colors as possible. What is the fewest number of colors you can use so that no areas that touch are the same color?  a. 2 * b. 3 c. 4 d. 5

Standard 4.5 Mathematical Processes

All students will use mathematical processes of problem solving, communication, connections, reasoning, representations, and technology to solve problems and communicate mathematical ideas.

While no additional big ideas, essential questions, or enduring understandings are listed for this standard, the mathematical processes are imbedded in the content-specific ideas, questions, and understandings delineated for the first four standards. References to the relevant processes can be found above.

4.5 A. Problem Solving

Descriptive Statement: *Problem posing and problem solving involve examining situations that arise in mathematics and other disciplines and in common experiences, describing these situations mathematically, formulating appropriate mathematical questions, and using a variety of strategies to find solutions. Through problem solving, students experience the power and usefulness of mathematics. Problem solving is interwoven throughout the grades to provide a context for learning and applying mathematical ideas.*

Areas of Focus	Comments and Examples
1. Learn mathematics through problem solving, inquiry, and discovery.	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
2. Solve problems that arise in mathematics and in other contexts. ▪ **Open-ended problems** ▪ **Non-routine problems** ▪ **Problems with multiple solutions** ▪ **Problems that can be solved in several ways**	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
3. Select and apply a variety of appropriate problem-solving strategies (e.g., "try a simpler problem" or "make a diagram") to solve problems.	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
4. Pose problems of various types and levels of difficulty.	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
5. Monitor their progress and reflect on the process of their problem solving activity.	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.

4.5 B. Communication

Descriptive Statement: *Communication of mathematical ideas involves students' sharing their mathematical understandings in oral and written form with their classmates, teachers, and parents. Such communication helps students clarify and solidify their understanding of mathematics and develop confidence in themselves as mathematics learners. It also enables teachers to better monitor student progress.*

Areas of Focus	Comments and Examples
1. Use communication to organize and clarify mathematical thinking. • **Reading and writing** • **Discussion, listening, and questioning**	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
2. Communicate mathematical thinking coherently and clearly to peers, teachers, and others, both orally and in writing.	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
3. Analyze and evaluate the mathematical thinking and strategies of others.	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
4. Use the language of mathematics to express mathematical ideas precisely.	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.

4.5 C. Connections

Descriptive Statement: *Making connections involves seeing relationships between different topics, and drawing on those relationships in future study. This applies within mathematics, so that students can translate readily between fractions and decimals, or between algebra and geometry; to other content areas, so that students understand how mathematics is used in the sciences, the social sciences, and the arts; and to the everyday world, so that students can connect school mathematics to daily life.*

Areas of Focus	Comments and Examples
1. Recognize recurring themes across mathematical domains (e.g., patterns in number, algebra, and geometry).	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
2. Use connections among mathematical ideas to explain concepts (e.g., two linear equations have a unique solution because the lines they represent intersect at a single point).	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
3. Recognize that mathematics is used in a variety of contexts outside of mathematics.	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
4. Apply mathematics in practical situations and in other disciplines.	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
5. Trace the development of mathematical concepts over time and across cultures (cf. world languages and social studies standards).	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
6. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.

4.5 D. Reasoning

Descriptive Statement: *Mathematical reasoning is the critical skill that enables a student to make use of all other mathematical skills. With the development of mathematical reasoning, students recognize that mathematics makes sense and can be understood. They learn how to evaluate situations, select problem-solving strategies, draw logical conclusions, develop and describe solutions, and recognize how those solutions can be applied.*

Areas of Focus	Comments and Examples
1. Recognize that mathematical facts, procedures, and claims must be justified.	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
2. Use reasoning to support their mathematical conclusions and problem solutions.	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
3. Select and use various types of reasoning and methods of proof.	**Instructional/Assessment Focus:** • This may be included in classroom enrichment activities at this grade level, but is more of a focus at secondary grade levels.
4. Rely on reasoning, rather than answer keys, teachers, or peers, to check the correctness of their problem solutions.	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
5. Make and investigate mathematical conjectures. ▪ Counterexamples as a means of disproving conjectures ▪ Verifying conjectures using informal reasoning or proofs	**Instructional/Assessment Focus:** • This may be included in classroom enrichment activities at this grade level, but is more of a focus at higher grade levels.

6. Evaluate examples of mathematical reasoning and determine whether they are valid.	**Instructional/Assessment Focus:** • This is more of a focus at secondary grade levels.

4.5 E. Representations

Descriptive Statement: *Representations refers to the use of physical objects, drawings, charts, graphs, and symbols to represent mathematical concepts and problem situations. By using various representations, students will be better able to communicate their thinking and solve problems. Using multiple representations will enrich the problem solver with alternative perspectives on the problem. Historically, people have developed and successfully used manipulatives (concrete representations such as fingers, base ten blocks, geoboards, and algebra tiles) and other representations (such as coordinate systems) to help them understand and develop mathematics.*

Areas of Focus	Comments and Examples
1. Create and use representations to organize, record, and communicate mathematical ideas. ▪ **Concrete representations (e.g., base-ten blocks or algebra tiles)** ▪ **Pictorial representations (e.g., diagrams, charts, or tables)** ▪ **Symbolic representations (e.g., a formula)** ▪ **Graphical representations (e.g., a line graph)**	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
2. Select, apply, and translate among mathematical representations to solve problems.	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
3. Use representations to model and interpret physical, social, and mathematical phenomena.	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.

4.5 F. Technology

Descriptive Statement: *Calculators and computers need to be used along with other mathematical tools by students in both instructional and assessment activities. These tools should be used, not to replace mental math and paper-and-pencil computational skills, but to enhance understanding of mathematics and the power to use mathematics. Students should explore both new and familiar concepts with calculators and computers and should also become proficient in using technology as it is used by adults (e.g., for assistance in solving real-world problems).*

Areas of Focus	Comments and Examples
1. Use technology to gather, analyze, and communicate mathematical information.	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
2. Use computer spreadsheets, software, and graphing utilities to organize and display quantitative information.	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
3. Use graphing calculators and computer software to investigate properties of functions and their graphs.	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
4. Use calculators as problem-solving tools (e.g., to explore patterns, to validate solutions).	**Instructional/Assessment Focus:** • Assessment of this CPI is within the context of one or more of the content CPIs 4.1 through 4.4.
5. Use computer software to make and verify conjectures about geometric objects.	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.
6. Use computer-based laboratory technology for mathematical applications in the sciences.	**Instructional/Assessment Focus:** • This CPI is largely an instructional CPI and is assessed within the context of one or more of the content CPIs 4.1 through 4.4.